

**Summer Math Review 2017**  
*For Section 12 AP Calculus AB*

- You ***should not use a calculator*** except for questions 9b, 10, 11, 14, and 17b.
- Work on a separate sheet of paper.
- Show all work.

1. Factor and simplify as completely as possible:

a.  $2x^2 - 10x + 12$

b.  $(x - 3)^3(2x + 1)^5 - (x - 3)^5(2x + 1)^4$

c.  $5x^2(2x - 3)^{\frac{1}{3}}(3x + 2)^{\frac{1}{2}} + 8x(2x - 3)^{-\frac{2}{3}}(3x + 2)^{\frac{3}{2}}$

d.  $9x^2 - 4y^2$

e.  $9x^2 + 4y^2$

f.  $8x^3 - 1$

2. Consider the graph of  $y = 2 - 3(x - 1)^2$ . What transformations must be applied to the graph of  $y = x^2$  to get this graph?

3. Sketch, and state the domain and range, of the following functions:

a.  $y = x^3$

b.  $y = \sqrt{x}$

c.  $y = \sqrt{x + 1}$

d.  $y = |x|$

e.  $y = e^x$

f.  $y = \ln x$

g.  $y = \sin x$

h.  $y = \sin^{-1} x$

i.  $y = \tan x$

4. A right circular cylinder has a volume of  $10 \text{ cm}^3$  and its height is twice the circumference of its base. Find the exact radius and height of the cylinder.
5. What are the amplitude, period, horizontal shift, and minimum and maximum values of

$$y = 3 \sin(2x - 1) + 5?$$

6. Explain the difference between  $4^{3^2}$  and  $(4^3)^2$ .
7. Give the exact value of (no calculators, remember!):

- a.  $\sin \frac{\pi}{3}$
- b.  $\tan \frac{3\pi}{4}$
- c.  $\sec \frac{7\pi}{6}$
- d.  $\cos \left(-\frac{\pi}{3}\right)$

8. Find the inverse of each of the following functions. If the inverse is not itself a function, describe how you could limit the original function's domain to make the inverse a function:

- a.  $y = 2x + 1$
- b.  $y = (x - 1)^2 + 3$
- c.  $y = 2^x$
- d.  $y = \sin(2x + 3) - 4$

9. If  $\sin \theta = \frac{1}{3}$ , then

- a. Find the exact values of the other 5 trigonometric functions at  $\theta$ . (Be careful, there might be more than one answer to some of them.)
- b. Use your calculator to approximate  $\theta$  in both degrees and radians. (Same warning as above.)

10. You want to know the height of a tree. Standing 100 feet away from the tree you measure an angle of elevation of  $40^\circ$ . How tall is the tree?
11. A truck starts east from an intersection at 1:00 traveling 80 km/hr. 15 minutes later a car starts north from the same intersection traveling 100 km/hr. How far apart are the two vehicles at 2:00? (Assume they continue at the same speed in the same direction at all times.)
12. Two missiles are traveling in opposite directions, headed towards each other. One is moving at a speed of 425 mph and the other is moving at 775 mph. How far apart are the missiles exactly 12 seconds before they collide?
13. Without a calculator, solve  $\sin x \tan x = \sqrt{3} \sin x$  for  $0 \leq x \leq 2\pi$ .
14. Use a calculator to solve  $e^x - 2 \sin x = 1$  for  $-\pi \leq x \leq \pi$ .
15. Calculate the following limits:
- $\lim_{x \rightarrow 3} \frac{x^2 - 3x + 2}{x^2 + x - 6}$
  - $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^2 + x - 6}$
  - $\lim_{x \rightarrow 0} \frac{\sin x}{2x}$
16. Find
- The slope of the line through (2,3) and (5,7).
  - An equation of this line.
  - An equation of the line parallel to the above line that passes through (-1,2).
  - An equation of the line perpendicular to the above line that passes through (-1,2).

17. Consider the function  $f(x) = 2 - 5 \cos(3x)$ .

- a. Is  $f$  an even function, an odd function, or neither?
- b. Use a calculator to find all zeros of  $f$  such that  $\frac{\pi}{2} \leq x \leq \pi$

18. Write each of the following in the form  $ax^b$  where  $a$  and  $b$  are rational constants to be found:

i.  $\frac{2}{x}$

ii.  $4\sqrt[3]{x}$

iii.  $\frac{4\sqrt{x}}{\left(\frac{4}{x}\right)^3}$

iv.  $\frac{3x^3 - 5x^2\sqrt{x}}{10\sqrt[4]{x^6} - \frac{6x^{-5}}{x^{-7}}}$

19. Find all asymptotes and holes, and describe the end behavior, of:

a.  $f(x) = \frac{(x-3)(x+2)}{(x+1)(x-3)}$

b.  $g(x) = \frac{x^2+4x+3}{x+1}$

20. Let the number  $A$  be 111111....11111 where the number of digits in  $A$  is 2003. Find the sum of the digits in the product  $2003A$ .